Analyzing Escher Tessellations

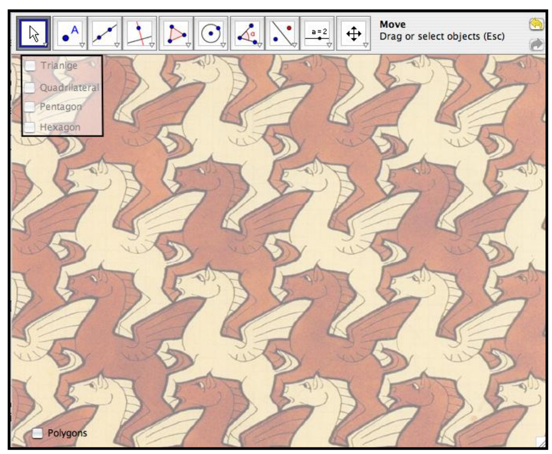
Introduction

M.C. Escher was the master of transforming polygonal tessellations into non-polygonal works of art. The connection between the symmetries of the underlying polygonal tessellations used by Escher, and the symmetries of his transformed tessellations can be traced back to the creation of his non-polygonal tiles. In this section, we will be using three dynamic worksheets “pegasus.html”, “dogs.html”, and “lizards.html” to explore these connections.

Pegasus

Open the dynamic worksheet *pegasus.html*. This worksheet is intended to help participants recognize and generalize a rule for finding the underlying polygonal tessellations that M.C. Escher used to create his works of art. An image of Escher’s *Pegasus* was imported into GeoGebra and has been locked to the background.

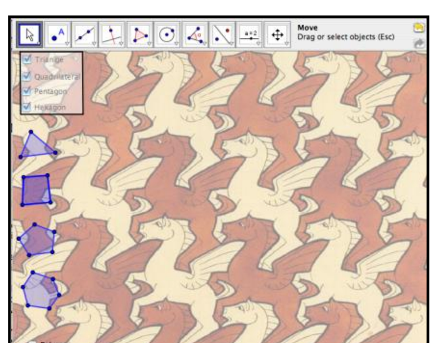
Give the participants a minute to examine and describe the features of the tessellation while you pose the following questions:



1. What polygon was used to create this tessellation?
2. Where are the vertices of these underlying polygons located?
3. What transformations could be used to tile the plane using one Pegasus?
4. What type of symmetry does this tessellation have?

After giving them a minute or two to think about these questions, spend about two minutes going over each one.

1. What polygon was used to create this tessellation?



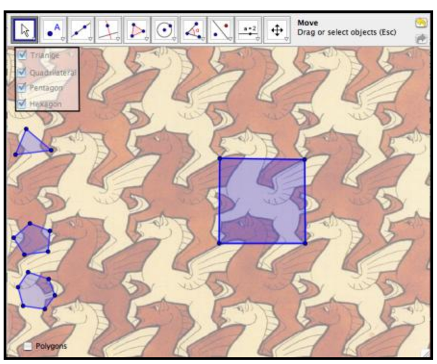
Click the check box for the triangle, quadrilateral, pentagon, and hexagon. Ask the participants which of the following polygons was used to create this tessellation.

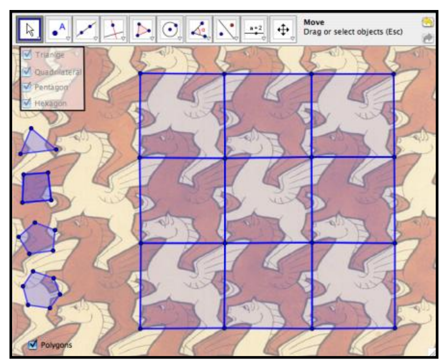
Entertain suggestions by dragging some polygons out into the drawing pad. Click in the middle of the polygon to do this. Then drag the vertices to give the polygon special features like parallel sides, congruent angles, etc. To transition into the next question, we might need to lead the participants to the correct answer. A square was used to create this tessellation. Some participants should recognize this, but often have a difficult time describing how they came to this conclusion. Parallel lines can be visualized if we move our eyes across the rows formed by the tops of the heads. Likewise, each Pegasus seems to stack on top of another one forming columns.

Pull the quadrilateral into the shape of the square and ask the participants where the vertices should be placed.

2. Where are the vertices of these underlying polygons located?

Have the participants visualize a tessellation of squares only. Ask them how many squares come together at each vertex. They should be able to see four. Have them now find the locations where four of the Pegasus tiles meet. Use the point tool to mark some of these locations.

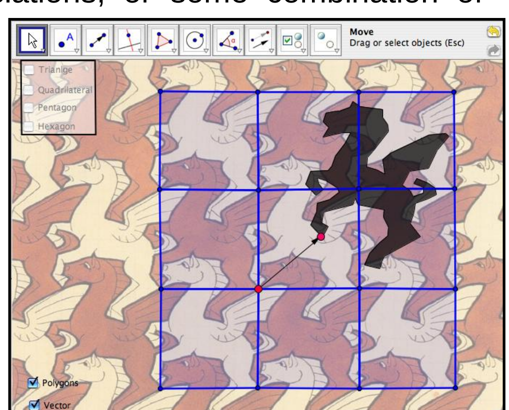




Select the "Polygon" check box at the bottom of the screen to bring forth a part of the underlying tessellation.

3. What transformations could be used to tile the plane using one Pegasus?

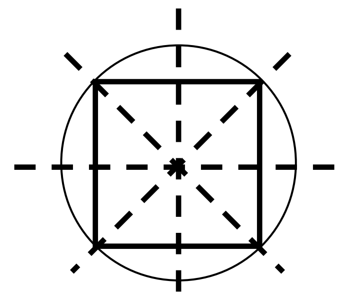
Could we use reflections, rotations, translations, or some combination of these transformations?



Every Pegasus is oriented in the same leftward facing direction. In addition, they could be viewed as stacked on top of each other. Through the use of color, they can be visualized moving in a diagonal fashion as well. All of these moves could be accomplished with translations.

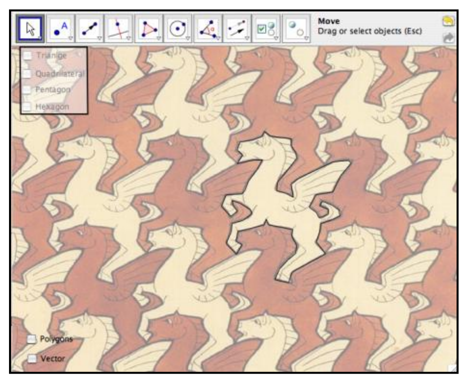
Select the “Vector” check box. A red dot will appear at a vertex in the underlying grid. Drag this dot and the Pegasus will move, for it is the head of a vector that translates the black Pegasus. Have the participants describe the translations that allow the Pegasus tile to fit into place.

4. **What type of symmetry does this tessellation have**?



What is a symmetry transformation? It is a transformation that leaves an object unchanged: “Something that does nothing.” Rotating a square, 90 degrees about its center, is a symmetry transformation. Have the participants visualize a square. Ask them: What types of transformations could be done to a square such that it would appear as if nothing had been done?

In the same respect, what transformation could be done to the entire tessellated plane such that it would appear as if nothing had changed?

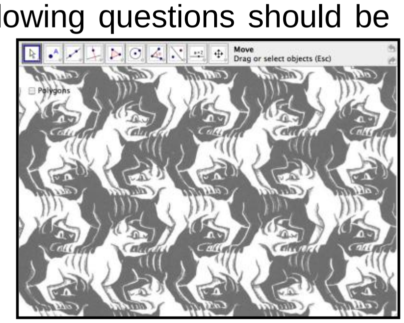


Have the participants visualize picking up the entire plane, doing something to it, and putting it back down such that everything looks the same.

Participants should state that the tessellation has translational symmetry. The translations from question 3 would cause the transformed plane to coincide with itself.

Dogs

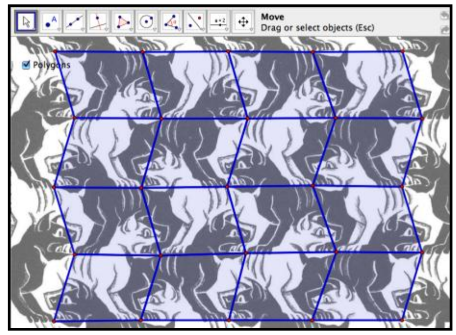
Open the dynamic worksheet *dogs.html* to provide participants with a quick example to practice what they just learned. The following questions should be asked to the participants:



1. What polygon was used to create this tessellation?

2. What type of symmetry does this tessellation have?

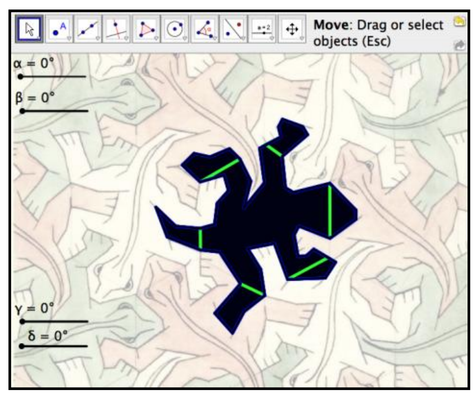
Use the point to place points at locations where ***more than two*** dogs meet. This is the general rule for finding the underlying lattice of polygons in Escher's non-polygonal tessellations. Why is it ***more than two***? Can we have a tessellation where there are two or less polygons coming together at a vertex?



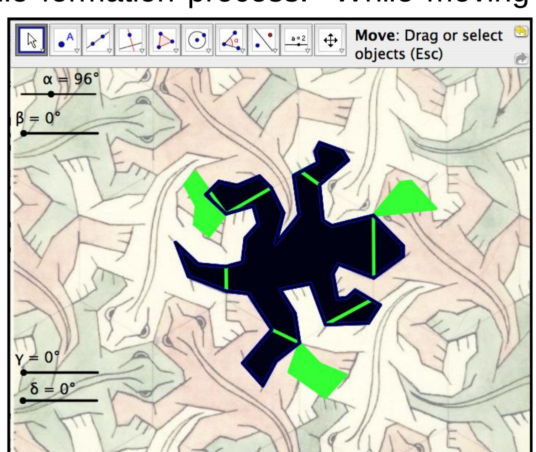
Select the "Polygon" check box. Notice that the parallelograms seem to changing direction in every row. As a result, the dogs are facing opposite directions. Ask the participants what transformation(s) would be needed to get from dog to dog. Translations and glide reflections will do the trick. These will also question number 2 above.

Lizards

Thus far, the participants have learned how to discover the underlying lattice of an Escher tessellation while discussing the symmetries inherent in each work of art. Participants have not discussed how the tiles are formed from their underlying polygon. While exploring the Lizards, participants will link the tile formation process to the symmetries of the tessellation.



Open the dynamic worksheet *lizards.html*. Without moving any sliders, allow the participants to state what the underlying polygonal tessellation is made from and what type of symmetries it has. They should respond that the tessellation is made from hexagons and has rotational and translational symmetries. If points are placed where any **three** lizards meet, the hexagons will emerge. The rotational symmetries can be seen at locations where the heads are next to each other. This can be found in other places as well. The lizards traveling in the same direction that are the same color will help the participants visualize the translational symmetry. To help the participants see the above symmetry transformations, move only sliders Alpha and Beta to their maximums and back again a few times to give insight into the tile formation process. While moving the sliders, ask and discuss the following questions:



1. Describe how the tile was formed from its underlying polygon? Use precise vocabulary.

2. Identify the vertices where the tile building transformations are taking place. Describe their location on the underlying polygon.

3. What transformation(s) could be used to move the tile into the spots of other lizards?

Make sure the following points are brought out while discussing the above questions:

1. **Describe how the tile was formed from its underlying polygon? Use precise vocabulary**.

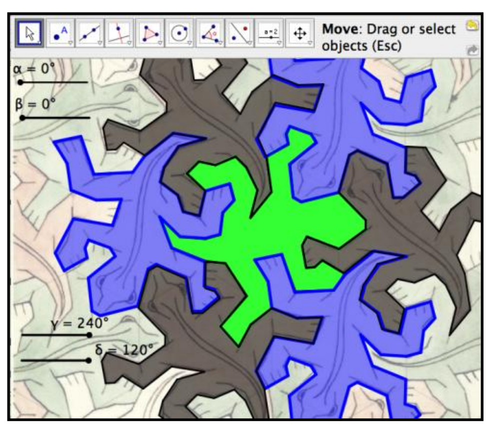
After moving sliders Alpha (α) and Beta (β), the participants should see that pieces were cut out of the hexagon. They were then rotated and attached back adjacent to remaining pieces of the hexagon's edges.

2. **Identify the vertices where the tile building transformations are taking place. Describe their location on the underlying polygon**.

All of the rotations are taking place around three alternating vertices in the hexagon. Make sure that the participants visually locate these three vertices. The pieces that move with slider Alpha (α) are easier to track than the ones which move with slider Beta (β).

3. **What transformation(s) could be used to move the tile into the spots of other lizards?**

Take input from the participants. Then, one at a time, move the sliders gamma (γ) and delta (δ) at the bottom of the page. Describe the transformations taking place using precise vocabulary.



In addition to the translational symmetry implied by the use of color, do the transformations above describe the symmetries of the tessellation?

Use the Alpha (α) slider to discuss how the movements of the blue lizards relate to creation of the original tile. The blue lizards will fit into the holes created by the Alpha (α) slider. Participants should see they follow the same path.

Use the Beta (β) slider to discuss how the movements of the black lizards relate to creation of the original tile. As with the tiles moved by the Alpha (α) slider, they tiles moved with the Beta (β) slider provide the holes for the black lizards to fit into.

Conclusion

In the activities above, participants discovered how to extract the underlying polygonal tessellation from which Escher created his works. In comparing the non-polygonal tile to its underlying polygon, keeping in mind the transformations used to create each tile, participants were able to link the symmetries of the tessellation to the tile bearing transformations. If time permits, return to Pegasus and Dogs to discuss how each of those tiles is created.